Non-linear matter power spectrum without non-linear dynamics modeling in $f(R)$ gravity

Cheng-Zong Ruan （阮承宗）

chzruan@mail.bnu.edu.cn

Department of Astronomy, Beijing Normal University

Supervised by Prof. Bin Hu & Tong-Jie Zhang

© 2019 CCNU-USTC Junior Cosmology Symposium

April 28, 2019

non-linear perturbation

- linear perturbation regime: density perturbation field

$$\delta(\boldsymbol{x}, t) \equiv \frac{\rho(\boldsymbol{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)} \ll 1$$

  linearized Einstein-Boltzmann equation and clean physics...

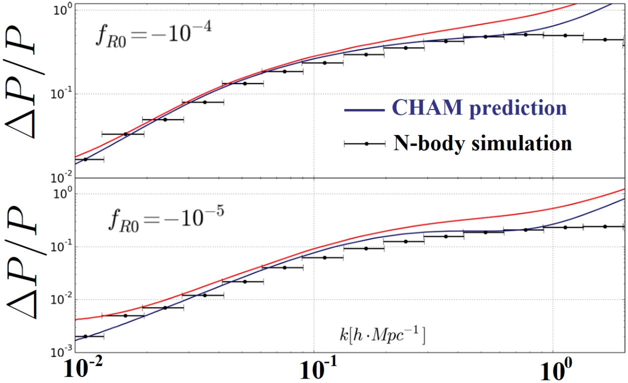
- non-linear regime: $\delta \gg 1$

  gravitationally bound, virialized systems: galaxy clusters (galaxies) are embedded in dark matter haloes (sub-haloes)

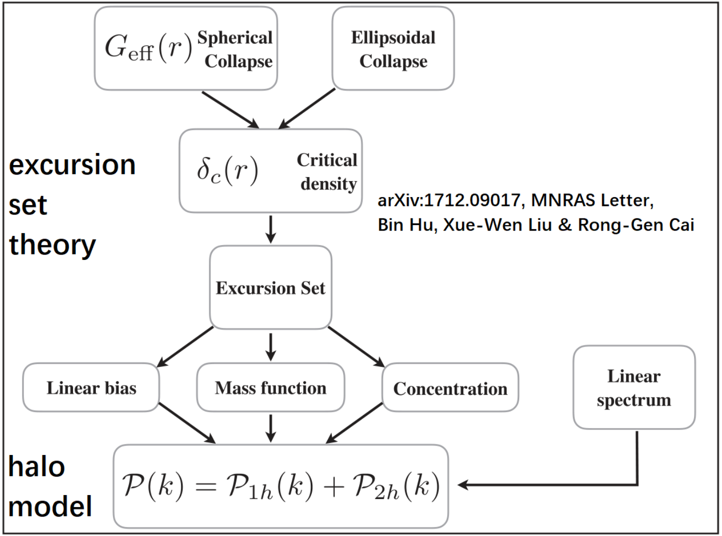
- researches on non-linear perturbation heavily rely on N-**body simulation**, which is *expensive, slow and only for one kind of parameter values at once*

CHAM [arXiv 1712.09017, by **B. Hu, X.-W. Liu & R.-G. Cai**]

- **CHAM**: a fast numerical s**C**reened **HA**lo **M**odel algorithm for modeling non-linear matter power spectrum in modified gravity (MG) theories
- slightly rely on N-body simulation results. Agree with $f(R)$ gravity N-body simulation power spectrum with $\lesssim 5\%$ deviation up to $k \sim 1\, h\,\mathrm{Mpc}^{-1}$



CHAM [arXiv 1712.09017]



CHAM assumption

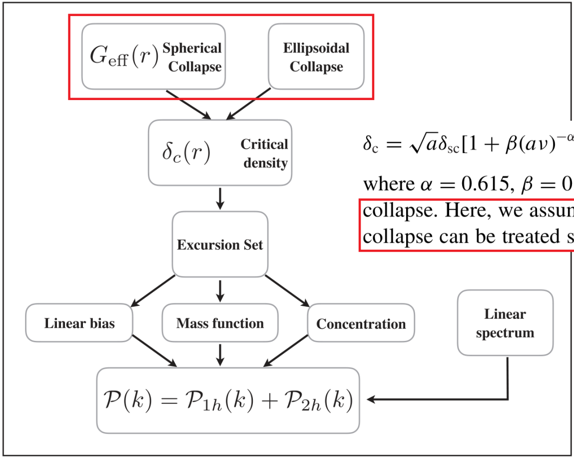


Figure 1. Flow chart of CHAM.

$$\delta_{\rm c} = \sqrt{a}\,\delta_{\rm sc}[1 + \beta(a\nu)^{-\alpha}],\ \nu \equiv \delta_{\rm sc}^2/S, \tag{10}$$

where $\alpha = 0.615$, $\beta = 0.485$, and $\delta_{\rm sc}$ is given by the spherical MG collapse. Here, we assumed that the effects of MG and ellipsoidal collapse can be treated separately.

... can we?

excursion set formalism

- excursion set formalism (also called the extended Press-Schechter theory): predict the collapsed DM haloes (deeply non-linear regime) by **partitioning the linear perturbation field** $\delta(\boldsymbol{x})$
- idea of W. Press & P. Schechter, ApJ, 187(425), 1974:

> mass fraction of haloes with $M > M_\star = \mathrm{Prob}\left[\delta_{\mathrm{smooth}}(\boldsymbol{x}; M_\star) > \delta_{\mathrm{c}}\right]$

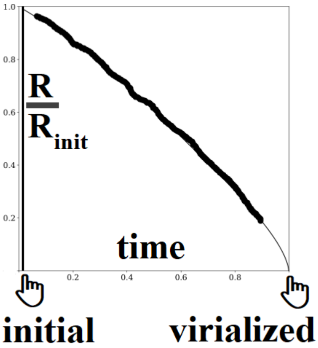
- $\delta_{\mathrm{smooth}}(\boldsymbol{x}; M_\star) \equiv \int \delta(\boldsymbol{x}')\, W(|\boldsymbol{x} - \boldsymbol{x}'|; M_\star) \mathrm{d}^3\boldsymbol{x}'$ is the smoothed linear overdensity field with filter size $R_\star \sim (M_\star/\bar{\rho}_{\mathrm{m}})^{1/3}$
- δ_{c}: **the collapse barrier**

collapse barrier δ_c

- **collapse barrier** $\delta_c \equiv \frac{D(a_0)}{D(a_{\rm init})}\delta_{\rm init}$**:** the linear overdensity **extrapolated** to virialized time (say, a_0) by using the linear growth function $D(a)$

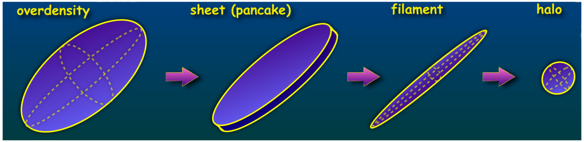
- the simplest case, spherical collapse in SCDM background cosmology: $\delta_{\rm sc} \equiv 1.686$

- collapse equation:
  - the Euler equation: $\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \sim -\nabla\Phi$
  - the Poisson equation: $\nabla^2\Phi \sim 4\pi G \underbrace{(1+\mathcal{F})}_{\text{MG effects}} \bar{\rho}_{\rm m}\delta$
  - mass conservation: $\bar{\rho}_{\rm m}(1+\delta)R^3 \sim M = \text{const.}$

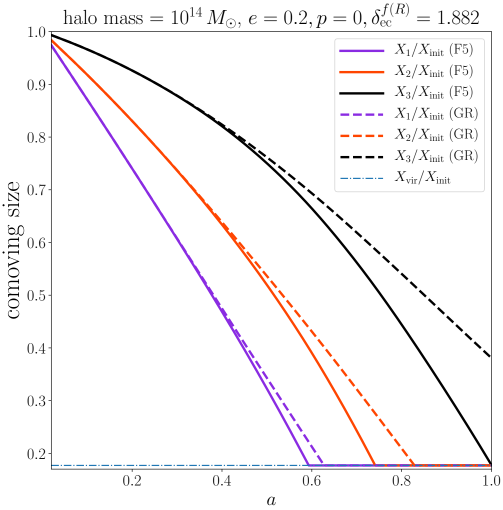


ellipsoidal collapse

- Zel'dovich approximation: an initially spherical overdensity would evolve into an **ellipsoid** with principal axes aligned with the eigen vectors of the tidal field $\nabla_i \nabla_j \Phi$



Frank Van den Bosch, lecture slides on ASTR 610: Theory of Galaxy Formation



ellipsoidal collapse

- a much better collapse barrier: ellipsoidal collapse (EC) barrier in ΛCDM background, solved by Sheth&Torman [astro-ph/9901122;9907024;0105113]

$$\text{valid in GR:} \quad \delta_c^{\mathrm{ST}}(\sigma) = \delta_{\mathrm{sc}}^{\mathrm{GR}}(M)\left\{1+\beta\left[\frac{\delta_{\mathrm{sc}}^{\mathrm{GR}}(M)}{\sigma(M)}\right]^{2\alpha}\right\}.$$

- in **CHAM**, the Sheth-Torman formula is used in $f(R)$ gravity as an assumption...

$$\text{extended to } f(R)\text{:} \quad \delta_c^{\mathrm{ST}}(\sigma) = \delta_{\mathrm{sc}}^{f(R)}(M)\left\{1+\beta\left[\frac{\delta_{\mathrm{sc}}^{f(R)}(M)}{\sigma(M)}\right]^{2\alpha}\right\}.$$

CHAM assumption

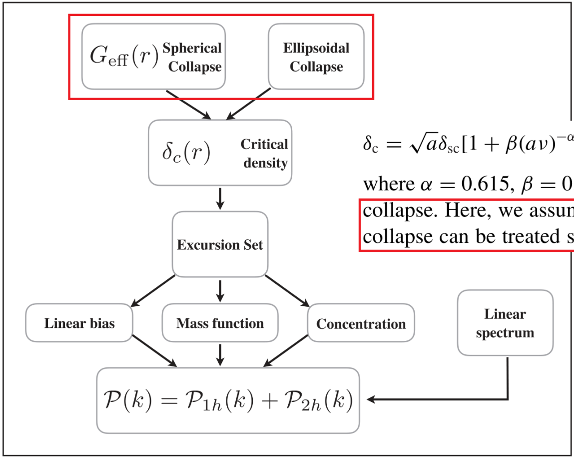


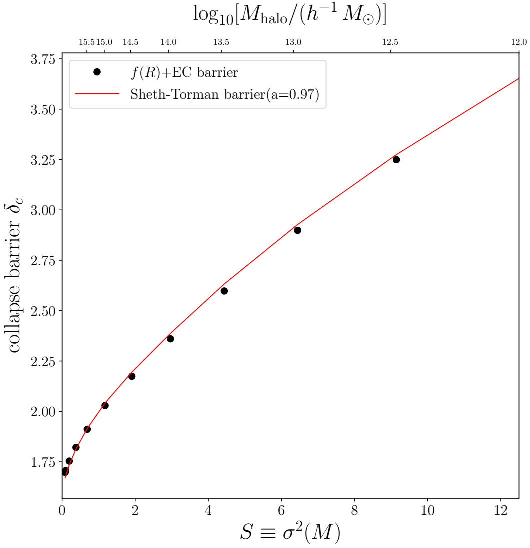
Figure 1. Flow chart of CHAM.

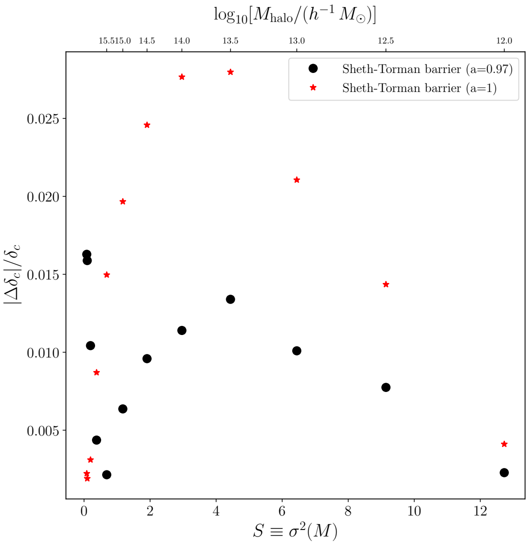
$$\delta_c = \sqrt{a}\delta_{sc}[1+\beta(a\nu)^{-\alpha}],\ \nu \equiv \delta_{sc}^2/S, \quad (10)$$

where $\alpha = 0.615$, $\beta = 0.485$, and δ_{sc} is given by the spherical MG collapse. Here, we assumed that the effects of MG and ellipsoidal collapse can be treated separately.

... can we?

yes we can (with $\lesssim 1.6\%$ difference)





collapse barrier comparison

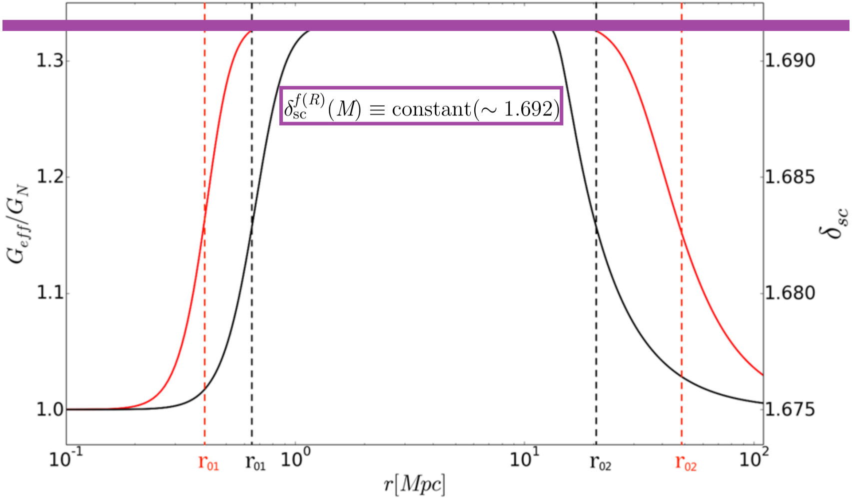
- the ellipsoidal collapse (EC) barrier in $f(R)$ gravity $\delta_{\rm ec}^{f(R)}$

$$\delta_{\rm ec}^{f(R)}(e, M, \delta_{\rm env}) \xrightarrow{\sigma(M)\approx\sigma_{\rm mp}=\sqrt{5}e\delta_{\rm ec}^{f(R)}} \delta_{\rm ec}^{f(R)}(\sigma, M) \xrightarrow{\sigma=\sigma(M)} \delta_{\rm ec}^{f(R)}\left[\sigma(M)\right] \ .$$

- $|\Delta\delta_c|/\delta_c \equiv \dfrac{|\delta_{\rm ec}^{f(R)} - \delta_c^{\rm ST}|}{\delta_{\rm ec}^{f(R)}} \lesssim 1.6\%$
- do not need to modeling the ellipsoidal collapse in $f(R)$ graivty since the Sheth-Torman formula is feasible

non-dynamical approximation (skipped)

- Non-dynamical approximation — $\delta_{\rm sc}^{f(R)}(M) \equiv \text{constant}(\sim 1.692)$



modeling non-linear matter power spectra

- (naively) revised **CHAM** code:
  `https://github.com/chzruan/cham_nondyn`

  (unstable so far, not the offical version)
- LSS surveys, such as *Euclid, LSST, WFIRST*, are aiming to measure the non-linear matter power spectrum up to **one percent** accuracy
- **CHAM** + non-dynamical approximation provides a fast prediction...

$$\text{Parameterization} = \left\{ \underbrace{\delta_{\rm sc}^{\rm non-dyn}, a_{\rm ST}}_{\text{excursion set formalism}} , \underbrace{H_0, \Omega_{\rm m0}, \ldots}_{\text{cosmological parameters}} \right\}$$